

Modelling and performance improvement of inverted pendulum using PID controller

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Abstract— Inverted pendulum is a standard problem in control system. This is a single input multiple output system having pendulum's angle and cart position as control parameters. In this user is able to dictate the position and velocity of the cart through the motor. It is a non-linear system, which can be treated as linear system, without much error and provides a good practice for prospective control engineers. This project presents the modelling of an inverted pendulum using differential equations for one degree of freedom. Its open loop performance on the basis of time response is observed which depicts that inverted pendulum is unstable as pendulum's angle diverges very rapidly. For stabilizing it, closed loop system is used. The purpose for this experiment was to test different algorithms used to control mechanical systems. By combining computer and electrical system with mechanical ones, mechanical systems can be controlled and provide responses that improve their ability to perform certain functions. The objective of the control system is to balance the inverted pendulum by applying force to the cart that the pendulum is attached to. This project proposes a procedure to control one of the parameter underneath the other. Using PID controller, the transient response and stability of the inverted pendulum is improved. The whole work presented in this report is simulated by using equations of motion using SIMULINK in MATLAB.

Keywords— Inverted pendulum, dynamic modelling, pid controller, state space model, MATLAB/SIMULINK.

I. INTRODUCTION

The inverted pendulum system is a standard problem in the area of control systems. They are often useful to demonstrate concepts in linear control such as the stabilization of unstable systems. Since the system is inherently nonlinear, it has also been useful in illustrating some of the ideas in nonlinear control. In this system, an inverted pendulum is attached to a cart equipped with a motor that drives it along a horizontal track. The user is able to dictate the position and velocity of the cart through the motor and the track restricts the cart to movement in the horizontal direction. Sensors are attached to the cart and the pivot in order to measure the cart position and pendulum joint angle, respectively.

II. MODELLING AND SYSTEM ANALYSIS

The cart with an inverted pendulum, shown below, is "bumped" with an impulse force, F . Determine the dynamic equations of motion for the system, and linearize about the pendulum's angle, $\theta = \pi$ (in other words, assume that pendulum does not move more than a few degrees away from the vertical, chosen to be at an angle of π).

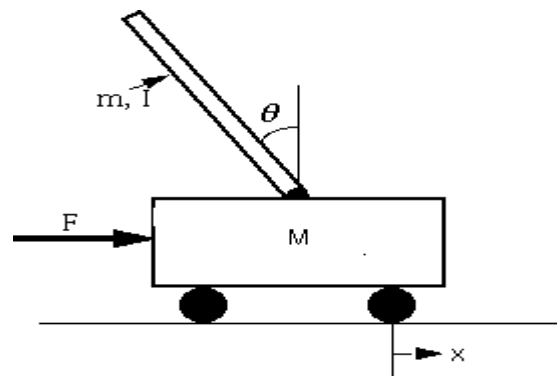


Fig 1 Cart with Inverted Pendulum

For this example, let's assume that

M	mass of the cart	0.5 kg
m	mass of the pendulum	0.2 kg
b	friction of the cart	0.1 N/m/sec
l	length to pendulum centre of mass	0.3 m
I	inertia of the pendulum	$0.006 \text{ kg} \cdot \text{m}^2$
F	force applied to the cart	
x	cart position coordinate	
theta	pendulum angle from vertical	

III. OPEN LOOP RESPONSE

In this system output does not depend upon input or we can say that output is not compared with the reference signal.

An open-loop controller, also called a non-feedback controller, is a type of controller that computes its input into a system using only the current state and its model of the system.

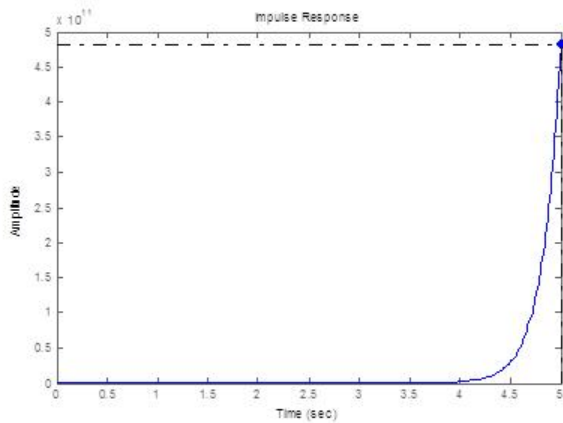


Fig 2 Open Loop response of pendulum's angle.

Analysis of time response give peak amplitude- 4.82e+011 in 5 sec as our requirement is not fulfilled so we have to control it by closed loop system.

The response is entirely unsatisfactory. It is not stable in open loop.

IV. CLOSED LOOP RESPONSE

A proportional–integral–derivative controller (PID controller) is a generic control loop feedback mechanism (controller) widely used in industrial control systems – a PID is the most commonly used feedback controller. A PID controller calculates an "error" value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error by adjusting the process control inputs.

The PID controller calculation (algorithm) involves three separate constant parameters, and is accordingly sometimes called three-term control: the proportional, the integral and derivative values, denoted P, I, and D. Heuristically, these values can be interpreted in terms of time: P depends on the present error, I on the accumulation of past errors, and D is a prediction of future errors, based on current rate of change. The weighted sum of these three actions is used to adjust the process via a control element such as the position of a control valve, or the power supplied to a heating element.

By tuning the three parameters in the PID controller algorithm, the controller can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error, the degree to which the controller overshoots the setpoint and the degree of system oscillation. Note that the use of the PID algorithm for control does not guarantee optimal control of the system or system stability.

Some applications may require using only one or two actions to provide the appropriate system control. This is

achieved by setting the other parameters to zero. A PID controller will be called a PI, PD, P or I controller in the absence of the respective control actions. PI controllers are fairly common, since derivative action is sensitive to measurement noise, whereas the absence of an integral term may prevent the system from reaching its target value due to the control action.

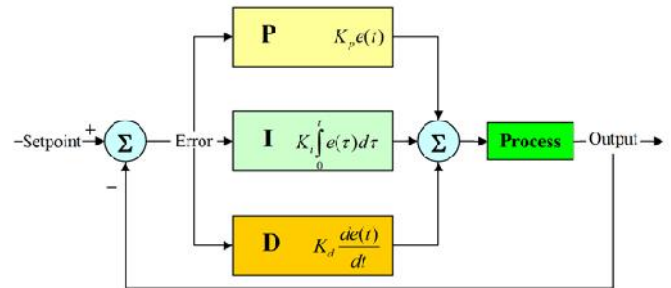


Fig 3 Closed Loop PID Controller

V. CLOSED – LOOP WITH DISTURBANCE

The control of this problem is a little different than the standard control problems, since we are trying to control the pendulum's position, which should return to the vertical after the initial disturbance, the reference signal we are tracking should be zero. The force applied to the cart can be added as an impulse disturbance. The schematic for this problem should look like the following.

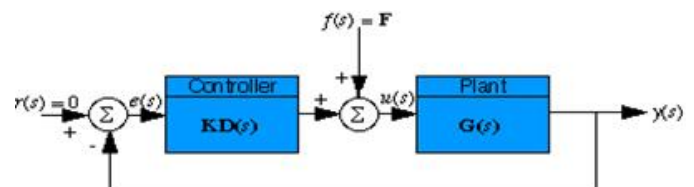


Fig 4 Closed Loop controller for Pendulum's position.

VI. TUNING

There are several methods for tuning a PID loop. The most effective methods generally involve the development of some form of process model, then choosing P, I, and D based on the dynamic model parameters. Manual tuning methods can be relatively inefficient, particularly if the loops have response times on the order of minutes or longer.

The choice of method will depend largely on whether or not the loop can be taken "offline" for tuning, and the

response time of the system. If the system can be taken offline, the best tuning method often involves subjecting the system to a step change in input, measuring the output as a function of time, and using this response to determine the control parameters.

Manual Tuning

If the system must remain online, one tuning method is to first set K_i and K_d values to zero. Increase the K_p until the output of the loop oscillates, then the K_p should be set to approximately half of that value for a "quarter amplitude decay" type response. Then increase K_i until any offset is corrected in sufficient time for the process. However, too much K_i will cause instability. Finally, increase K_d , if required, until the loop is acceptably quick to reach its reference after a load disturbance. However, too much K_d will cause excessive response and overshoot. A fast PID loop tuning usually overshoots slightly to reach the setpoint more quickly; however, some systems cannot accept overshoot, in which case an over-damped closed-loop system is required, which will require a K_p setting significantly less than half that of the K_p setting causing oscillation.

Ziegler-Nichols Method

Another heuristic tuning method is formally known as the Ziegler–Nichols method, introduced by John G. Ziegler and Nathaniel B. Nichols in the 1940s. As in the method above, the K_i and K_d gains are first set to zero. The P gain is increased until it reaches the ultimate gain, K_u , at which the output of the loop starts to oscillate. K_u and the oscillation period P_u are used to set the gains as shown:

Ziegler–Nichols method

Control Type	K_p	K_i	K_d
P	$0.50K_u$	-	-
PI	$0.45K_u$	$1.2K_p/P_u$	-
PID	$0.60K_u$	$2K_p/P_u$	$K_pP_u/8$

We should get the following velocity response plot from the impulse disturbance:

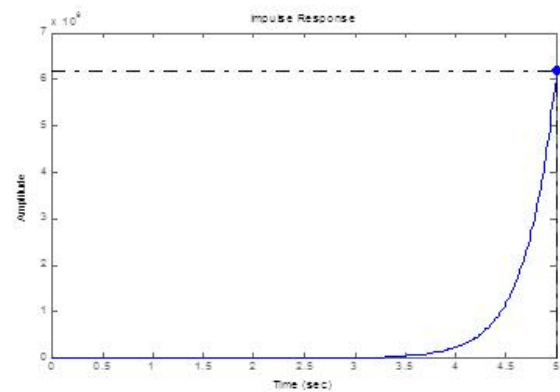


Fig 5 Closed Loop Response with $k_p=1, k_d=1, k_i=1$.

Analysis of time response give settling time within 2% and peak amplitude 6.19×10^6 which is not desired. This response is still not stable. Let's start by increasing the proportional control to the system. Increase the K_p variable to see what effect it has on the response. If you set $K_p=100$, and set the axis to axis $([0, 2.5, -0.2, 0.2])$, we should get the following velocity response plot:

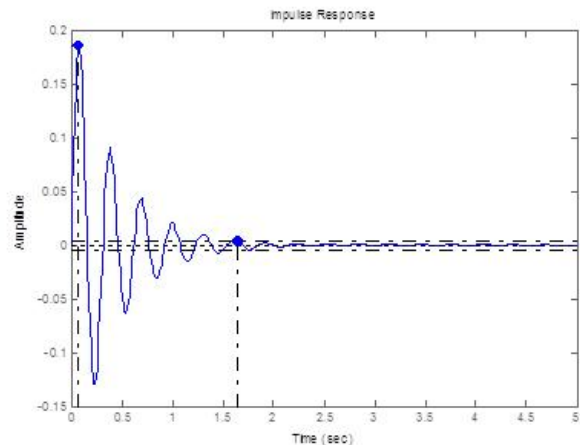


Fig 6 Closed Loop Response with $k_p=100, k_d=1, k_i=1$.

Analysis of time response give settling time of the response is determined to be 1.64 sec, which is less than the requirement of 5sec. Peak response, however is larger than the requirement of 0.05 radians. The settling time is acceptable at about 2 seconds. Since the steady-state error has already been reduced to zero, no more integral control is needed. We can remove the integral gain constant to see for ourself that the small integral control is needed. The overshoot is too high, so that must be fixed. To alleviate this problem, increase the K_d variable. With $K_d=20$. We should now see the following velocity response plot:

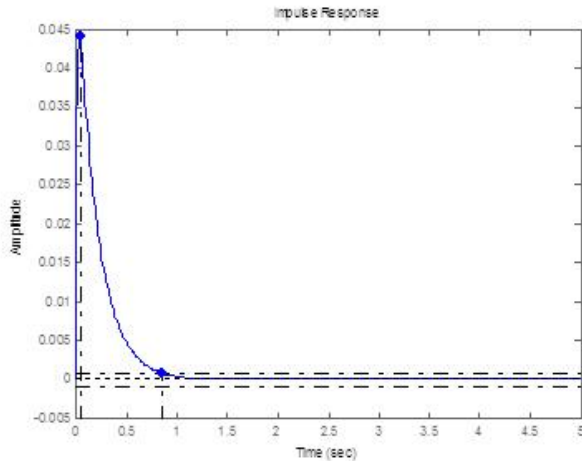


Fig 7 Closed Loop Response with $k_p=100$, $k_d=20$, $k_i=1$.

Analysis of time response give peak amplitude 0.0442 and settling time 0.844. As we can see, the overshoot has been reduced so that the pendulum does not move more than 0.05 radians away from the vertical. All of the design criteria have been met, so no further iteration is needed.

VII. CART POSITION

The block representing the position was left out because that variable was not being controlled. It is interesting though, to see what happening to the cart's position when the controller for the pendulum's angle is in place. To see this we need to consider the actual system block diagram:

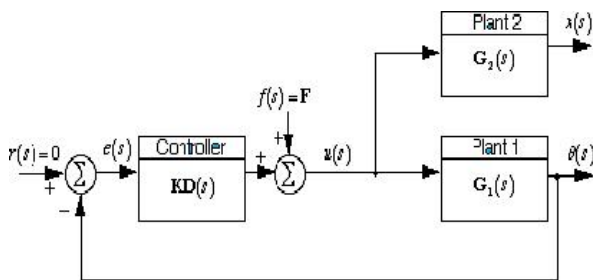


Fig 8 Closed loop For Cart Position

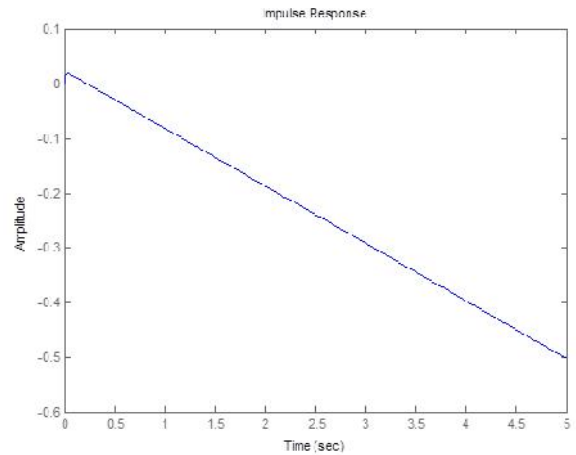


Fig 9 Closed loop Response of Cart Position

As we can see, the cart moves in the negative direction with a constant velocity. So although the PID controller stabilizes the angle of the pendulum, this design would not be feasible to implement on an actual physical system.

VIII. CONCLUSIONS

The in this study, a PID controller is designed and employed for controlling pendulum's angle and cart's position. The model here is to developed a balance between pendulum's angle and cart position so that the pendulum exactly fall over it.

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